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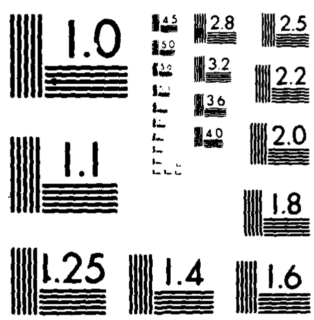

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FOR MAJORITY RULE

by

Herve Raynaud

Technical Report No. 331
March, 1981

A REPORT OF THE
CENTER FOR RESEARCH ON ORGANIZATIONAL EFFICIENCY
STANFORD UNIVERSITY

Contract ONR-N00014-79-C-0685, United States Office of Naval Research

THE ECONOMICS SERIES
INSTITUTE FOR MATHEMATICAL STUDIES IN THE SOCIAL SCIENCES
Fourth Floor, Encina Hall
Stanford University
Stanford, California
94305

PARADOXICAL RESULTS FROM INADA'S CONDITIONS
FOR MAJORITY RULE*

by

Herve Raynaud**

1. Characterization of the profiles which follow the not-in-the middle condition, but not the bipartition condition.

In an earlier paper (Raynaud [1979]), I answered a question asked by Morton [1966] in his review of a famous note by Inada [1964]. This was done by exhibiting a simple counter-example to the supposed equivalence between the bipartition and the not-in-the middle condition.

I am now able to give a much more complete answer to the question, an answer which allows the solution of enumerating problems.

In what follows:

- X denotes a finite set of alternatives or objects $\{a,b,\ldots y,z\}$;
- For any set S, $|S|$ denotes the cardinality of X;
- E (or E(X)) denotes a profile, i.e., a sequence of N total orders $\theta_1, \theta_2, \ldots, \theta_N$ on the $|X| = n$ considered objects; each θ_i is called an individual order
- If $Y \subseteq X$, E(Y) will denote the sequence given restrictions of the θ_i's to the objects in Y.

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*This work was supported in part by Office of Naval Research Grant ONR-N00014-79-C-0685 and in part by the Fulbright Exchange Program and the Center for Research in Organizational Efficiency, Stanford University Thanks to K.J. Arrow and P. Coughlin for their very careful review of this paper.

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Definition 1: A profile follows the not-in-the-middle condition if, for any triple T of objects of X, there exists one of the three objects which is never ranked second in any of the orders of $E(T)$.

Definition 2: A profile follows the bipartition condition if, for all Y included in X, there exists a bipartition of Y in Y_1, Y_2 such that any individual order $\theta_i(Y)$ can always be written $\theta_i(Y_1)\theta_i(Y_2)$ or $\theta_i(Y_2)\theta_i(Y_1)$.

We shall say that any individual order on Y can be written under the shape Y_1Y_2 or under the shape Y_2Y_1.

Many people have proven that these conditions ensure the transitivity of the majority method, and that the second one implies the first (e.g., Romero [1978]).

Definition 3: A profile E is said to contain configuration K if there are four object, a, b, c, d, in X and two individual orders θ and θ' in E such that $\theta(a,b,c,d) = abcd$ or $dcba$ and $\theta'(a,b,c,d) = bdac$ or $cadb$.

Theorem 1: If a profile satisfies the not-in-the-middle condition and if $|X| = 4$, two cases are possible:

(i) It satisfies the bipartition condition and does not contain configuration K.

(ii) It contains configuration K and does not satisfy the bipartition condition.

Proof: The exhaustive enumeration (cumbersome but easily reduced to 32 cases) of the different possible maximal profiles shows that they can be divided in two classes.

(i) A class of profiles all satisfying clearly a bipartition condition.

(ii) A class of profiles containing configuration K.

It is easy to see that this class does not satisfy any bipartition condition by checking the fact that the four bipartitions made of one singleton and one triple and the three bipartitions made of two pairs of objects cannot describe any of the four possible of individual orders necessarily appearing in a profile containing configuration K.

Hence, if a profile contains configuration K (satisfying or not the not-in-the-middle condition), it is not a bipartition. If it is, on the contrary, a bipartition, it cannot contain configuration K.

Proposition 1: If a profile on n objects contains configuration K, it cannot satisfy the bipartition condition.

Proof: Trivial: If a, b, c, d, are the four objects considered for configuration K, take $Y = \{a,b,c,d\}$; then $\exists\, y$ such that $E(Y)$ contains no bipartition Y_1, Y_2 such that any individual orders be written $E(Y_1)E(Y_2)$ or $E(Y_2)E(Y_1)$. And this is precisely the contrary of the bipartition condition.

Proposition 2: Let E be a profile on four objects containing configuration K and following the not-in-the-middle condition. Let abcd(1), dcab(2), bdac(3), cadb(4) be the four votes corresponding to

configuration K. If E contains only two or three of these four votes, E can be completed by the lacking one, without loss of the not-in-the-middle condition.

Theorem 2: If a profile on n objects follows the not-in-the-middle condition then two possibilities only can occur:

(i) E contains configuration K and does not satisfy the bipartition condition.

(ii) E satisfies the bipartition condition and does not contain configuration K.

If E contains configuration K, it is the object of Proposition 1 to show that it does not satisfy the bipartition condition. What remains to be proven is that if it does not satisfy the bipartition condition then it contains configuration K.

We know that this is already true for four objects and the proof will be done by induction; it needs two introductory Lemmas.

Definition 4: E being a profile on X, a "sequential bipartition" will consist of a sequence of sets S, X_1, X_2, X_{11}, X_{12}, X_{21}, X_{22}, ... such that

(i) If $|X| > 1$, all $\theta_i(X)$ can be written $\theta_i(X_1)\theta_i(X_2)$ or $\theta_i(X_2)\theta_i(X_1)$.

(ii) If $|X_i| > 1$, all $\theta_i(X_i)$ can be written $\theta_i(X_{i1})\theta_i(X_{i2})$ or $\theta_i(X_{i2})\theta_i(X_{i1})$

⋮

(iii) If $|X_{i,j,\ldots,\ell}| > 1$, all $\theta_i(X_{i,j,\ldots,\ell})$ can be written

$\theta_{i\ldots\ell}(X_{i\ldots\ell,1})\theta_{i\ldots\ell}(X_{i\ldots\ell,2})$ or $\theta_{i\ldots\ell}(X_{i\ldots\ell,2})$

$\theta_{i\ldots\ell}(X_{i\ldots\ell,1})$

$\vdots$

Lemma 1: If E, profile on X, has a sequential bipartition, then it follows the bipartition condition (and conversely).

The proof is straightforward. Consider $Y \subseteq X$. There exists a smallest set among $X, X_1, X_2, X_{11}, X_{12}, X_{21} \ldots$ which includes Y. Let $X_{ij\ldots\ell}$ be this set. Then $Y \cap X_{i\ldots\ell,1} = Y_1$ and $Y \cap X_{i\ldots\ell,2} = Y_2$ are such that any $\theta_i(Y)$ can be written $\theta_i(Y_1)\theta_i(Y_2)$ or $\theta_i(Y_2)\theta_i(Y_1)$. The converse is trivial.

Let us suppose now that the theorem is true for any $|X| = 4, 5 \ldots (n-2), (n-1)$ and let us show that it is true too for $|X| = n$.

For this, let us consider a profile E on n objects following the not-in-the-middle condition and not the bipartition condition. If T is a triple of objects of X, E(T) follows the not-in-the-middle condition and the bipartition condition (which is trivially true for any triple following any not-in-the-middle condition).

Lemma 2: If a profile E(X) follows the not-in-the-middle condition, but not the bipartition condition, there exists a subset x of X such that

(i) E(x) satisfies the not-in-the-middle and not the bipartition condition,

(ii) but for any strict subset y of x then $E(y)$ satisfies the not-in-the-middle and the bipartition condition.

Proof:

(1) Delete progressively the elements of X, in any order. In step i, you delete x_i and the remaining objects of X constitute the set $X_i = \{x_{i+1},\ldots,x_n\}$. In that sequence of deletions, you will necessarily reach a step i_o such that $E(X_{i_o})$ does not follow the bipartition condition and $E(X_{i_o+1})$ does (this is clear because $E(X_{n-3}) = E\{x_{n-2},x_{n-1},x_n\}$ follows the bipartition condition, as does any profile on three objects respecting the not-in-the-middle condition).

(2) Then check whether all strict subset of $E(X_{i_o})$ follows the bipartition condition. If yes, the lemma is proven.

(3) If no, then there exists an x_α such that $E(X_{i_o} - x_\alpha)$ does not follow the bipartition condition. Exchanging the indices of x_α and x_{i_o+1}, the new X_{i_o+1} is such that $E(X_{i_o+1})$ does not follow the bipartition condition.

Hence deleting progressively elements from the new X_{i_o+1}, you will reach an X_{i_1} such that $E(X_{i_1})$ does not follow the bipartition condition but $E(X_{i_1+1})$ does. You can, then, loop the algorithm at step (2).

The algorithm is necessarily finite because $|X_{i_o}| > |X_{i_1}| > |X_{i_2}| \ldots > 0$ and will prove the lemma because $E(T)$, T being any triple of elements in X, follows the bipartition condition.

The lemma can be expressed under a formulation more efficient for what follows: if $E(X)$ satisfies the not-in-the-middle and not the bipartition condition, then there exists a subset x of X and an element λ in $X - x$ such that $x \cup \lambda$ does not satisfy the bipartition and all its strict subsets satisfy the bipartition condition.

Let us consider χ. Since χ is a strict subset of $\chi \cup \lambda$, from the hypothesis, all the individual orders in $E(\chi)$ can be written under the shape X, Y or under the shape Y, X, ($\{X,Y\}$ being a certain partition of χ).

If X^{λ} (resp. Y^{λ}) denotes a sequence including all the objects of X (resp. Y) plus λ, λ being <u>not</u> an extreme in the sequence, introducing the object λ can produce five "types" of votes, namely

(1) λXY or $YX\lambda$

(2) $X^{\lambda}Y$ or YX^{λ}

(3) $X\lambda Y$ or $Y\lambda X$

(4) XY^{λ} or $Y^{\lambda}X$

(5) $XY\lambda$ or λYX

A. Let us suppose now that $E(\chi \cup \lambda)$ contains votes of one type only. If it is type 1 or 2 or 3, then $X \cup \lambda$ and Y make a bipartition which indicates, according to the lemma and the induction hypothesis, that the bipartition would hold in $E(X)$, which is impossible. If it is type 4 or 5, then X and $Y \cup \lambda$ give such a bipartition.

B. Let us suppose now that $E(\chi \cup \lambda)$ contains votes of exactly two types. If the pairs of types were (1,2), (1,3), (2,3), the bipartition $\{(X \cup \lambda),Y\}$ would ensure the contradiction. If the pairs of types were (4,5), (4,3), (5,3) the symmetrical $\{X,(Y + \lambda)\}$ would do. (1,5) has a clear bipartition into λ and $(X \cup Y)$.

Since (1,4) and (2,5) are symmetrical, the same reasoning will hold for both of them. Let us consider (1,4). For $\{Y \cup \lambda\}$, from the induction hypothesis, there exists a bipartition such that all the votes in $E(Y \cup \lambda)$ can be written under the shape $\{Y_1 \cup \lambda\}Y_2$ or under the shape $Y_2\{Y_1 \cup \lambda\}$. Hence, the votes in 1 can be written under the shape

$\lambda X Y_1 Y_2$ or $Y_2 Y_1 X \lambda$ and votes in 4 will be $X\{Y_1 \cup \lambda\}Y_2$ or $XY_2\{Y_1 \cup \lambda\}$ or $\{Y_1 \cup \lambda\}Y_2X$ or $Y_2\{Y_1 \cup \lambda\}X$.

From this, $\{Y_1 \cup \lambda\}Y_2X$ or $XY_2\{Y_1 \cup \lambda\}$ has to appear among the votes (or a bipartition in Y_2, $\{X \cup Y_1 \cup \lambda\}$ would be clear). Let it be the first. Then, if y_1 denotes an element of Y_1 on the other side of λ from X in the votes of the shape $\{Y_1 \cup \lambda\}Y_2X$, y_2 an element of Y_2, x an element in X, one finds necessarily in E:

(a) a vote $\lambda x y_1 y_2$ or $y_2 y_1 x \lambda$

(b) a vote $y_1 \lambda y_2 x$

which is part of the configuration K on those four letters. The same reasoning would hold for votes in 4 of the shape $XY_2\{Y_1 \cup \lambda\}$. The remaining case, that is to say (2,4) is the more confusing. The proof, after that, will become more easy.

(2,4) means that:

(a) one vote $\{X^\lambda\}Y$ or $Y\{X^\lambda\}$ occurs and

(b) one vote $X\{Y^\lambda\}$ or $\{Y^\lambda\}X$ occurs.

There is a bipartition of $X \cup \lambda$ in $X_1 \cup \lambda$, X_2 which will allow to write the votes $\{X^\lambda\}Y$ and $Y\{X^\lambda\}$ in the shapes $\{X_1 \cup \lambda\}X_2Y$ or $X_2\{X_1 \cup \lambda\}Y$ or $Y\{X_1 \cup \lambda\}X_2$ or $YX_2\{X_1 \cup \lambda\}$.

For the same reason Y^λ is bipartitioned in $Y_1 \cup \lambda$ and Y_2, which develops (2,4) as follows:

2. $\{X_1 \cup \lambda\}X_2Y_1Y_2$ or $X_2\{X_1 \cup \lambda\}Y_1Y_2$

or $Y_2Y_1\{X_1 \cup \lambda\}X_2$ or $Y_2Y_1X_2\{X_1 \cup \lambda\}$

4. $X_2X_1\{Y_1 \cup \lambda\}Y_2$ or $X_2X_1Y_2\{Y_1 \cup \lambda\}$

or $\{Y_1 \cup \lambda\}Y_2X_1X_2$ or $Y_2\{Y_1 \cup \lambda\}X_1X_2$

In order not to have a bipartition in X_2, one of the votes

$$A: \{X_1 \cup \lambda\}X_2Y_1Y_2 \text{ or } Y_2Y_1X_2\{X_1 \cup \lambda\} \text{ has to be voted.}$$

In order not to have a bipartition in Y_2, one of the votes

$$B: X_2X_1Y_2\{Y_1 \cup \lambda\} \text{ or } \{Y_1 \cup \lambda\}Y_2X_1X_2 \text{ has to be voted.}$$

If now, x_1 denotes an element of X_1 extremal in the votes A and y_1 an element of Y_1 extremal in the votes B; if x_2 and y_2 denote current elements of X_2 and Y_2 respectively. One shall encounter: in A: $x_1 \ldots \lambda \ldots x_2 \ldots y_1 \ldots y_2 \ldots$ or $\ldots y_2 \ldots y_1 \ldots x_2 \ldots \lambda \ldots x_1$; in B: $\ldots x_2 \ldots x_1 \ldots y_2 \ldots \lambda \ldots y_1$ or $y_1 \ldots \lambda \ldots y_2 \ldots x_1 \ldots x_2 \ldots$ which exhibits configuration K in y_2, y_1, x_2, λ.

From this, three votes at least are necessary, each one in a different type. Let us suppose that three types occur at least. These cases will, according to the sub-cases, show a bipartition, or the non-respect of the not-in-the-middle condition.

(1) If the profile is (1,2,3), it shows the bipartition $\{X \cup \lambda, Y\}$. If the profile strictly contains (1,2,3), it contains at least one vote 4 or 5. If it is 4, then $\exists\, x \in X$, $\exists\, y \in Y$ such that the triple $\{x\lambda y\}$ does not respect the non-in-the-middle condition. If it is 5, then none of the triples $\{x\lambda y\}$ with $x \in X$ and $y \in Y$ respects the not-in-the-middle condition. Case (3,4,5) is symmetrical and follows the same reasoning.

(2) For all the other possible triples it is always trivial that the not-in-the-middle condition will not be respected for at least one triple.

I want, incidentally, to quote some of the very remarquable properties of configuration K. The first is that its four votes constitute a latin square:

- a b c d
- b d a c
- c a d b
- d c b a

What is more, if you rank its four orders in such a way that the first letter in each line has a rank in the first line which is equal to the number of the line, then the rows are identical to the columns. Four symmetrical designs can be obtained

a b c d	b d a c	c a d b	d c b a
b d a c	d c b a	a b c d	c a d b
c a d b	a b c d	d c b a	b d a c
d c b a	c a d b	b d a c	a b c d

Conjecture: There is a nice economical or sociological interpretation of this mathematical piece of baroque-which is not pure rococo (Sen [1979]).

2. Enumeration Problems

An important purpose of this research is to prove that, from the individual point of view, the freedom in voting is not greater in Inada's

conditions than in the original Black's condition (Black [1958]). In other words, that the conditions which are only expressed on triples (like Arrow's single peakedness (Arrow [1963]) and the complementary conditions of Inada) do not allow more individual freedom than Black's condition (which is based on a reference order for the alternatives).

How can one count the maximum number of different votes in a profile following single peakedness, single cavedness and not-in-the-middle condition?

The two first cases are in G. Kohler's thesis (Kohler [1978])--published in French. As this student of mine has decided to abandon research, after his Ph.D., I will present here the result.

Let us consider the so-called single peakedness condition: on any triple T of objects in X, one of the three objects is never ranked last in $E(T)$.

Proposition 3: Single peakedness is equivalent to the following condition C: $\forall\ Y \subseteq X$, two objects of Y at most can be encountered in the last rank of $E(Y)$.

Proof: $C \Rightarrow$ single peakedness. Trivial. For any triple T, take $Y = T$. If two objects at most are last in $E(T)$, then one at least is never last!

Let us suppose now that we have single peakedness and not condition C. There exists $Y \subseteq X$ such that at least 3 objects of Y appear in the last rank of the orders of $E(Y)$. Let us consider a triple T of

such three objects. In $E(T)$ the three objects of T will be encountered in the last rank-which is in contradiction with the hypothesis.

Theorem 3: The maximum number of votes in a profile following single peakedness (or single cavedness) is 2^{n-1} if n is the number of alternatives.

Proof: Let us consider the case of single peakedness. From Romero [1978], the same result will hold for single cavedness. According to Proposition 3, there are at most two different objects in the last position, say y and z. If one considers the votes ending in y for instance, the penultimate object (being the last for $X - y$) can be one of only two objects in $X - y$ and so on.

We obtain, hence an upper bound of 2^{n-1}. It is known that this upper bound is reached in the Blackian particular case.

The case of the "not-in-the-middle" condition had resisted Kohler, had been thought to be identical to bipartition by Inada, and was more mysterious. However:

Proposition 4: The maximum number of different votes in a profile following the bipartition condition is 2^{n-1}, where n denotes the number of alternatives.

Proof: The result is trivial for $n = 2$. Let us suppose it true from 2 to $n - 1$, and let us prove it for n. If X has n objects, $n > 2$, $\{X_1,X_2\}$ is a bipartition of X in non-empty sets such that all the votes can be written under the shape X_1X_2 or X_2X_1. $|X_1| = m$,

$|X_2| = p$, $m + p = n$. The different votes in $E(X_1)$ are 2^{m-1}, in $E(X_2)$, 2^{n-1} at most. Hence, the different votes in $E(X)$ are at most

$$2 \times 2^{m-1} \times 2^{p-1} = 2^{n-1}$$

In order to count the maximal number for the not-in-the-middle condition, I first tried a technique directly similar to Kohler's technique. This gave the following

Proposition 5: The not-in-the-middle condition is equivalent to the following: C': $\forall\, Y \subseteq X$, $|Y| \geq 3$ there exists at least a pair of alternative $x, y \in Y$ not to be ranked simultaneously as the extremals in $E(X)$.

Proof: Let Y be any triple. $C' \Rightarrow$ not-in-the-middle for Y. Not-in-the-middle $\Rightarrow C'$: consider the negation of C' i.e. $\exists\, Y \subseteq X$, $|Y| > 3$ such that all pairs xy of alternatives are extremals in at least one of the votes of $E(Y)$.

Consider then a triple $\{x,y,z\}$ of alternatives in Y. There exists at least in E:

θ' such that in $\theta'(Y)$ one has $x \ldots y \ldots z$ or $z \ldots y \ldots x$

θ'' such that in $\theta''(Y)$ one has $x \ldots z \ldots y$ or $y \ldots z \ldots x$

θ''' such that in $\theta'''(Y)$ one has $z \ldots x \ldots y$ or $y \ldots x \ldots z$

and the not-in-the-middle condition cannot be respected.

<u>Proposition 6</u>: If a profile follows the not-in-the-middle condition, the set of pairs of alternatives which can be ranked simultaneously as extremes in an individual order can be identified with the set of edges of a bipartite graphe, the vertices of which are the elements of X.

<u>Proof</u>: Let us consider a profile E on n objects, satisfying the not-in-the-middle condition. Let us associate with E the graph G, the vertices of which being the elements of X, and the edges of which joining vertices which are simultaneously extremes in one individual order at least. Then, clearly, G is triangle free (i.e., does not contain any circuit made of 3 edges only). We shall prove by induction and reduction ad absurdum that G is odd circuit free (i.e., does not contain any circuit made of an odd number of edges).

Let us suppose that the theorem is true from 3 to 2p - 1 objects, in other words that any G is free from circuits of odd order smaller or equal to 2p - 1 and that, on the contrary, there exists a profile E with a corresponding G which contains a circuit (α...tuvxyzabcdef...α) of order 2p + 1. Let us consider an individual order in E having "a" and "z" as extremals. We can always suppose that "a" is the first and "z" the last. Then, the penultimate can be neither x (for, in the set, {α...tuvxabcd...α}, one would close an odd circuit of order smaller than 2p + 1), nor u, ... nor e, nor c. For the same reason, the second object in the considered vote is none of these objects. For this reason, the pair making the second and the penultimate can only

be made of elements in b, d, f ... t, v, y. But if such a pair is chosen, the graph corresponding to the vertices of part of the circuit including the pair, and all the vertices which are on the side of the circuit which does not include a and z, closes an odd circuit of order smaller that $2p + 1$. Being without odd circuit, G is bipartite according to the classical result.

A more direct approach, with the help of Theorem 2 proved to be more efficient. It is clear that any profile satisfying the not-in-the-middle condition can be obtained by introducing additional alternatives progressively into individual orders, always respecting the condition. (The reverse could be obtained by deleting progressively the alternatives from E(X)).

<u>Proposition 7</u>: In the progressive process of constructing a profile on n objects following the not-in-the-middle condition, a new alternative introduced in E(Y) can only be introduced at two different ranks for each vote in E(Y).

<u>Proof</u>: Let us suppose, on the contrary that, for one vote, x could be introduced in three different ranks, $i < j < k$. In the vote in E(Y) the object in rank i is y, the object in rank j is z. The three orders including x and coming from the considered vote are then

...x...y...z...

...y...x...z

...y...z...x...

Hence, the triple x, y, z does not follow the not-in-the-middle condition, which is contrary to the hypothesis.

<u>Theorem 4</u>: Let E be a profile satisfying the not-in-the-middle condition. If it follows a bipartition condition, it has a maximum of 2^{n-1} different votes. If it does not follow a bipartition condition, it has a maximum of 2^{n-2} different votes.

<u>Proof</u>: In the first case, the result comes from Proposition 4. According to the Theorem 2, in the second case, the profile E is such that it counts at least four objects forming configuration K. But if a profile on four objects follows the not-in-the middle condition and not the bipartition condition and contains all of the four votes of the configuration K, then no other vote can be added without destroying the not-in-the-middle condition. Any vote is going to begin by a letter and configuration K is completely symetrical: hence it is enough to prove that no vote beginning by an "a" can be added. Those votes are

a b d c 1

a c b d 2

a c d b 3

a d c b 4

a d b c 5

(they, of course, have to be different from abcd).

In K, b is the only one to never be in the middle of $\{b,c,d\}$. Hence, votes 2 and 5 are inacceptable. In the same way c is the only one to never be in the middle of $\{a,b,c\}$ in K, hence votes 3 and 4 are inacceptable. In the same way, d is the only one to never be in the middle of $\{a,d,c\}$ in K, hence vote 1 is inacceptable.

For this reason, let us consider a, b, c, d, objects of X such that E contains configuration K, and let us look at the different votes in $E(\{a,b,c,d\})$. They are at most four. Adding a new element λ of X to the set $\{a,b,c,d\}$, I shall have in $E(\{abcd\lambda\})$ at most 2×2^2 ... using this progressive process to build up $E(X)$, one sees clearly that each different vote in one step can at most give two different votes at the following step. From this comes the result.

The conclusion of this paper however paradoxical is simple: Arrow-Black's and Inada's conditions on triples do not allow more diversity individual orders than the more restrictive looking Black's condition with a reference order. If one considers that the culture imposes one of these conditions to the individual, and that a measure of his freedom consists in the number of different opinions he is allowed to have (in order to have the right to vote), no more is offered by the conditions on triples only.

References

Arrow, K.J. [1963], Social Choice and Individual Value, New York: Wiley.

Black, D. [1958], The Theory of Committees and Elections, Cambridge University Press.

Inada, K.Y.C. [1964], "A Note on the Simple Majority Decision Rule," Econometrica, 32, 1964.

Kohler, G. [1978], Choix Multicritere et Analyse Algebrique des Donnees Ordinales. Thesis, Grenoble, France.

Morton, G. [1966], Math. Review No. 1119.

Raynaud, H. [1979], "A Propos de Quelques Conditions de Transitivite de la Methode Majoritaire Bien Connues," Colloque Aide a la Decision et Jeux de Strategies: Aspects Multi-Criteres, Bruxelles Institut des Hautes Etudes.

Romero, D. [1978], Variations sur l'Effet Condorcet. Thesis Grenoble.

Sen, A. [1979], Last Words of Concluding Remarks in Social Choice Theory, a paper prepared for the Handbook of Mathematical Economics (to appear ed. K.J. Arrow and M. Intriligator).

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